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Giant quantum oscillations of a second kind

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Abstract. Giant quantum oscillations (GQO) of the coefficient of ultrasonic absorption by normal metals subject to a magnetic field during a $2\frac{1}{2}$-order phase transition (PT-$2\frac{1}{2}$) is discussed. Only the simplest case of the appearance (disappearance) of a spheroidal Fermi sheet associated with the phase transition is considered. For electron energies much higher than the threshold energy, the absorption coefficient is found to show GQO which are not only periodic in inverse magnetic field (the Gurevich–Skobov–Firsov (GSF) oscillations) but also in electron energy (with the magnetic field kept constant) or the size of the cavity (Z) created during the phase transition.

1. Introduction

Generally, sound absorption is believed to be a lattice phenomenon. But, at low enough temperatures $T \ll T_D$, where T_D is the Debye temperature, ultrasonic absorbtion by good metals is to a large extent dominated by electronic mechanisms [1, 2].

At such ultrahigh frequencies ($kl \gg 1$, where k is the sound wave vector and l is the mean free path), the process of sound absorption becomes a quantum phenomenon [3, 4] in which individual phonons of the incident ultrasonic wave are absorbed by conduction electrons of the metal. Conservation of energy and momentum principles indicate that, since normally the Fermi velocity of the electrons is much higher than that of the sound wave, the electrons that effectively absorb sound energy are only those moving practically in a plane of constant phase of the sound wave at an angle close to 90° to the sound wave vector $\boldsymbol{k}$ or at an angle of the order of s/v_F from the constant phase plane, where s is the velocity of the sound wave in the metal and v_F is the Fermi velocity of the conduction electrons.

The above statements also hold true in the presence of a magnetic field (arbitrarily directed along the z-axis) which in this case is assumed to be only strong enough to wipe out the smoothing effect of temperature. The magnetic field is not expected to alter the dispersion law for the conduction electrons either, which, for the sake of simplicity, is considered to be quadratic–isotropic. This work does not include the effect of spin splitting. Interested readers may refer to the work by Rodriguez [4].

If we neglect the excitation of electrons due to transverse electric fields induced by the incident ulrasonic wave [5] and the possible electron–electron interaction, the ultrasonic absorption coefficient will be given by [1]

$$\Gamma_H = \Gamma_0 \frac{\hbar\omega}{4k_B T} \sum_n \int \mathrm{d}p_z \frac{\delta(p_{z0} - p_z)}{\cosh^2[(\epsilon_{n,p_z} - \epsilon_F)/k_B T]} \tag{1}$$

where ω is the Larmor frequency, n is the quantum number related to the Landau levels, and Γ_0 is the absorption coefficient in the absence of magnetic field, the exact expression of which was first derived by Akheizer and co-workers [6]. Also, ϵ_F is the electron Fermi

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energy, p is the electron quasimomentum and T is the temperature. The quantity $p_{z0} \sim ms$, where m is the electron mass, is very small (since k is parallel to the applied magnetic field), and hence has little effect on the overall characteristic of Γ_H [1]. k_B is the Boltzmann factor.

Near the minimum or maximum of a given energy band the density of states shows a root-type singularity, termed a Van Hove singularity (VHS) [7]. VHS is characteristic of all energy values at which the topology of the constant-energy surface changes. Actually, a minimum is a point where a new Fermi sheet appears and a maximum is where an existing Fermi sheet disappears.

Deep inside a given energy band we also find at least two critical values of energy ϵ_c [7] having a spectral density of the type

$$\nu(\epsilon) = \nu_0(\epsilon) + \delta\nu(\epsilon) \tag{2}$$

where ν_0 is a smooth function of energy and $\delta\nu$ is singular and is described by the relation

$$\delta\nu(\epsilon) = \begin{cases} 0 & \epsilon_F < \epsilon_c \\ \dfrac{\sqrt{2}V}{\pi^2\hbar^3}\sqrt{m_1 m_2 m_3}(\epsilon_F - \epsilon_c)^{1/2} & \epsilon_F \geqslant \epsilon_c. \end{cases} \tag{3}$$

V is the volume of the crystal lattice. The electron effective masses m_1, m_2, and m_3 are all positive. The discontinuity of ν at $\epsilon = \epsilon_c$ is characteristic of all saddle points of ϵ, which themselves are the direct result of the periodic nature of the lattice [7]. If there is some continuously varying parameter, like pressure or impurity, in the course of whose variation $\epsilon_F - \epsilon_c$ passes through zero, then the singularity of the spectral density and the dynamics of electrons near the critical surface leads to peculiar anomalies in the thermodynamic and kinetic properties of the electron gas in the metal [8], the ultrasonic absorption coefficient Γ being one.

The phase transitions associated with such critical values of energy are termed PT-$2\frac{1}{2}$. The usual appearance (or disappearance) of Fermi sheets accompanies these transitions [9–13]. In this paper we shall be concerned only about the simplest case of the appearance (or disappearance) of a spheroidal sheet.

2. Giant quantum oscillations

The case of electronic phase transition associated with the appearance (disappearance) of a spheroidal Fermi sheet can be included in (1) by introducing the appropriate dispersion law ϵ_{n,p_z} valid in the vicinity of the critical point ϵ_c. In the assumption that the dispersion law for the electrons remains quadratic–isotropic even in the presence of a magnetic field, the electron energy may be written as

$$\epsilon_{n,p_z} = \epsilon_c + \hbar\omega(n + \tfrac{1}{2}) + \frac{p_z^2}{2m}. \tag{4}$$

The Fermi surface of the metal is also assumed to have no interband sections. As a result, the ultrasonic absorption coefficient given in (1) will take the form

$$\Gamma_H = \Gamma_0 \frac{\hbar\omega}{4k_B T} \sum_n \left[\cosh^2\left(\frac{\hbar\omega(n+\frac{1}{2})}{2k_B T} - \frac{(Z - Z^*)}{2k_B T}\right)\right]^{-1} \tag{5}$$

where

$$Z^* = \frac{p_{z0}^2}{2m}$$

and

$$Z = \epsilon_F - \epsilon_c.$$

$Z^* \sim ms^2/2$ is a threshold of electron energy at which the anomaly of the absorption coefficient should begin.

Under proper limits, (5) actually reduces to well known results. For example, in the limit as H approaches zero, the summation can very well be replaced by integration which after some algebraic work may be shown to take the form

$$\Gamma_H = \frac{\Gamma_0}{2}\left(1 + \tanh\frac{Z - Z^*}{2k_B T}\right). \tag{6}$$

Subjecting (6) to a further constraint on the temperature ($T \to 0$), we get

$$\Gamma_H = \begin{cases} \Gamma_0 & Z \geqslant Z^* \\ 0 & Z < Z^*. \end{cases} \tag{7}$$

The above expression is a well known result first derived by Davydov and Kaganov [11].

Let us now consider the effect of finite values of magnetic field and temperature on the jump predicted in (7). As should be anticipated, those conduction electrons with Z close to the threshold value $Z^*(Z^* \gg k_B T)$, due to the random orientation of their Fermi momentum and also due to the adverse effect of the magnetic field, will not have the necessary translational motion for them to stay in constant phase with the incident longitudinal (k parallel to the applied magnetic field) ultrasonic wave. Therefore, the large majority of these electrons will not be able to significantly participate in the sound-energy absorption process. Consequently, in this particular case, the ultrasonic absorption coefficient Γ_H is expected to be relatively small. Actually, for $Z \simeq Z^*$, (5) can be shown to reduce to the simple form

$$\Gamma_H = \Gamma_0 \alpha \exp(-\alpha) \tag{8}$$

where

$$\alpha \sim \frac{\hbar\omega}{2k_B T}.$$

Since $\hbar\omega \gg k_B T$, we then find from (8) that the ultrasonic absorption coefficient at $Z \simeq Z^*$ is indeed exponentially small. The result given in (8) also shows that both magnetic field and temperature smear out the jump predicted in (7). In fact, it can easily be verified from (8) that, for both $\alpha \ll 1$ and $\alpha \gg 1$, $\Gamma_H \ll \Gamma_0$. Γ_H approaches a maximum for $\hbar\omega \simeq 2k_B T$.

For electron energies less than the threshold value $Z^*(0 < z < Z^*)$, the ultrasonic absorption coefficient once again is expected to be insignificantly small, but will not be at its absolute minimum as predicted in (7). In fact, in this case, the ultrasonic absorption coefficient can be shown to monotonically increase from $\Gamma_{\min} \sim \exp(-Z^*/2k_B T)\Gamma_{\max}$, at

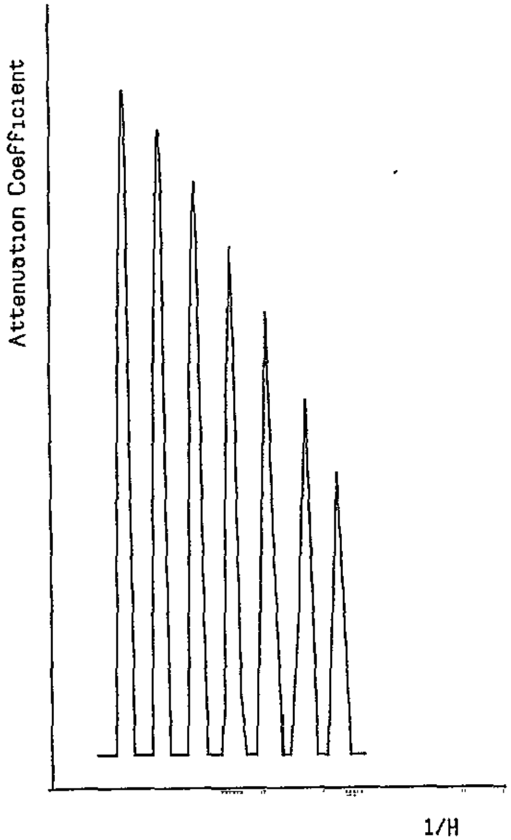

Figure 1. The periodicity of the absorption coefficient Γ_H in inverse magnetic field as predicted by (9) and (10). At very low temperatures ($\sim$ 1.5 K), it should be possible to observe oscillations of Γ_H for applied magnetic fields larger than 10 kG.

$Z \sim 0$, to Γ_{max}, where Γ_{max} is the absorption coefficient given in (8). The sound absorption in this particular range of Z is primarily due to random scattering.

The most dramatic and interesting region of electron energy is that in which $Z \gg Z^*$ ($Z^* \gg k_B T$). In this particular case, the ultrasonic absorption coefficient Γ_H given in (5) can be rewritten as a sum of two parts:

$$\Gamma_H = \Gamma_0 + \tilde{\Gamma} \tag{9}$$

where the second term on the right-hand side is oscillatory with respect to both H^{-1} and Z (figures 1 and 2). This term is given by

$$\tilde{\Gamma} = \Gamma_0 \left(\frac{4\pi^2 k_B T}{\hbar\omega}\right) \sum_{k=1}^{\infty} (-1)^k k \cos\left(\frac{2\pi k\beta}{\hbar\omega}\right) \operatorname{cosech}\left(\frac{2\pi^2 k k_B T}{\hbar\omega}\right) \tag{10}$$

where

$$\beta = Z - Z^*.$$

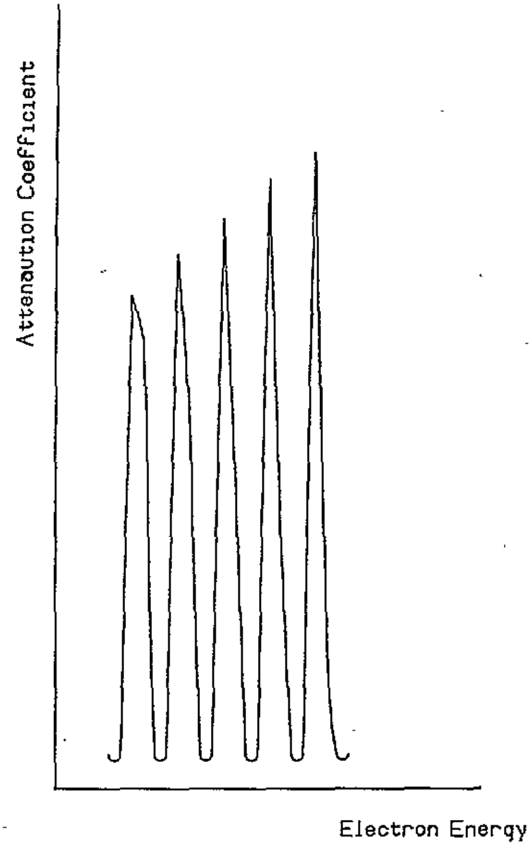

Figure 2. The periodicity of the absorption coefficient Γ_H in electron energy ϵ (for arbitrary constant applied field) as predicted by (9) and (10). Oscillations of Γ_H are expected to be seen for electron energy much larger than the threshold value, $Z^* = ms^2/2$, where s is the speed of a compressional wave in the material and m is the electron mass.

The periods of oscillation in inverse magnetic field and Z are

$$\Delta(1/H) = \frac{e\hbar}{Zmc} \tag{11}$$

and

$$\Delta(Z) = \hbar\omega \tag{12}$$

respectively.

For the set of conditions $Z - Z^* \gg \hbar\omega \gg k_B T$, first given by GSF [1], $\Gamma_{H,\max}$ and $\Gamma_{H,\min}$ can be determined from (5) to be

$$\Gamma_{H,\max} = \Gamma_0 \frac{\sqrt{3}}{2\pi} \frac{\hbar\omega}{k_B T} \exp\left[\left(\frac{\sqrt{6}}{\pi}\right) \frac{\beta}{k_B T}\right] \tag{13}$$

and

$$\Gamma_{H,\min} \sim \Gamma_0 \tag{14}$$

respectively. From (13) and (14) then follows

$$\frac{\Gamma_{H,\max}}{\Gamma_{H,\min}} \gg 1 \tag{15}$$

which shows that the oscillations of the ultrasonic absorption coefficient given by (9) and (10) are indeed gigantic. These oscillations are termed GQO [1].

The set of conditions for observing GQO ($Z - Z^* \gg \hbar\omega \gg k_B T$) indicate that the phenomenon is primarily due to those relevant states with relatively high quantum numbers, a regime where quantum and semiclassical approaches seem to intersect. A classical analysis of these conditions will then show that observation of GQO is only possible provided the de Broglie wavelength for the electron is much smaller than the characteristic size of the orbit of the electron in the magnetic field. This on the other hand means that, in order to observe GQO, the applied magnetic field H should be much smaller than the crystal field H_a, where

$$H_a \sim \frac{c\hbar}{|e|a^2} \tag{16}$$

and a is the inter-atomic distance. Here, e is the electronic charge.

3. Discussion and concluding remarks

As briefly pointed out in section 1, the thermodynamic, transport and magnetic properties (specific heat, resistivity etc) of ordinary metals (Cu, Ag etc) largely depend upon the geometry of the Fermi surface. Generally, a change in the topology of the Fermi surface *will lead to a singularity in the density of states, ν, and consequently to sharp discontinuous* changes (anomalies) of the above-mentioned properties.

It is known that high pressures reduce the anisotropy of most of the properties mentioned in the preceding paragraph. Gaidukov and Itskevich (GI) have reported that the connectivity of the Fermi surface of zinc decreases with increasing hydrostatic compression [14]. Since *typically Fermi surfaces of layered structures are corrugated cylinders, it is then not unusual* for these surfaces to gradually deform into closed ones (approximate spheres) under strong hydrostatic compression, even though the number of electrons in the conduction band remains the same [2].

Generally, the values of ϵ_c at which such transitions take place are located sufficiently far from the Fermi energy, and therefore a relatively high hydrostatic pressure may be required to get to these singular points (or establish $\epsilon_F - \epsilon_c = 0$). The extent to which the *lattice is deformed by the applied pressure, in such cases, depends upon the compressibility* of the individual specimen. Assuming a 5–10% lattice deformation, the hydrostatic pressure we are looking at may be anywhere between 10 and 200 kbar. GI have predicted that the connectivity of the Fermi surface of Zn would be broken by hydrostatic pressures of about 30 kbar [14].

The transitions we are considering here (the transitions that take place at $\epsilon_F = \epsilon_c$) are totally different from other transitions, such as first-order phase transitions, that could also occur under the influence of high pressure. The transitions that take place at $\epsilon_F = \epsilon_c$ (PT-$2\frac{1}{2}$) are the only ones that are associated with the singularity of density of states, which normally results in the appearance (or disappearance) of Fermi sheets. If these two transitions occur in a single experiment, the order in which they take place is of no important consequence basically because of the large difference in the time scales involved. It must be possible to observe the anomalies related to PT-$2\frac{1}{2}$ briefly even under metastable conditions.

A large number of high-pressure experiments on solids have shown that with increasing pressure the resistivity of many metals shows sharp discontinuous rises [15]. For example, barium transits to the FCC close-packed phase under a hydrostatic pressure of about 58 kbar. One would not then expect, in this metal, transitions involving more efficient arrangement of atoms as many of the phase transitions are customarily explained. The normal behaviour which one would expect is a continuous decrease in resistance with pressure due to a stiffening of the lattice. But on the contrary, for this metal, the resistance increases with increasing pressure in the lower-pressure part of the region of interest, and at about 140+ kbar the resistance shows a discontinuous sharp rise. We find no conclusive explanation given for this particular phenomenon anywhere. If it is not a first-order phase transition, what could it be? It is very likely that, in this case, the metal may be experiencing a $2\frac{1}{2}$-order phase transition.

Going back to the case of ultrasonic absorption, without the presence of a magnetic field, the phase transition that occurs at $\epsilon_F = \epsilon_c$ would result in a sharp discontinuous change of the ultrasonic absorption coefficient Γ, as first predicted by Davydov and Kaganov [9–12] (refer to (7)). A further increase in hydrostatic pressure will eventually force the newly created Fermi sphere (cavity) to grow. This is because of the reciprocal relationship existing between real space and momentum space written very simply as

$$k_F \sim \frac{1}{d} \tag{17}$$

where k_F is the Fermi wave vector and d is the lattice dimension. Note that, in the free-electron model, as the atomic volume decreases under pressure, the Fermi surface grows but remains spherical. This is only true if the metal is isotropically compressed. In the presence of a small but constant quantizing field, the continuous growth of the size of the Fermi sphere brought about by a slowly and smoothly increasing hydrostatic pressure would then result, as will be fully discussed later in this section, in giant oscillations of the ultrasonic absorption coefficient Γ_H (figure 2).

From (9) and (10), we actually derived two kinds of giant oscillation whose periods are given by (11) and (12) respectively. The latter defines the period of the oscillations just mentioned in the above paragraph. For the sake of convenience these oscillations will be referred to as 'GQO of the second kind' hereafter. The quantum oscillations whose period is given by (11) are not new to the scientific community. They were first predicted by GSF [1]. These oscillations are periodic in H^{-1} (figure 1), as clearly shown in (11), and are essentially caused by the broadening of the gap between the Landau levels as the applied magnetic field is continuously increased. This in turn causes the oscillations of the density of states at the Fermi level, and eventually the oscillations of the ultrasonic absorption coefficient Γ_H. The period given in (11) is very closely related to the de Haas–van Alphen period [16, 17]:

$$\Delta(1/H) = 2\pi\hbar/cS(\epsilon, p_z) \tag{18}$$

where $S(\epsilon, p_z)$ is the extremal cross-section of the Fermi sphere. As is well known, measurements of the periods of the de Haas–van Alphen oscillations give estimates of the various cross-sectional areas of the Fermi surface [18].

As pointed out earlier in this section, the quantum oscillations that are introduced as new in this work, 'GQO of a second kind', occur in the presence of a small but constant

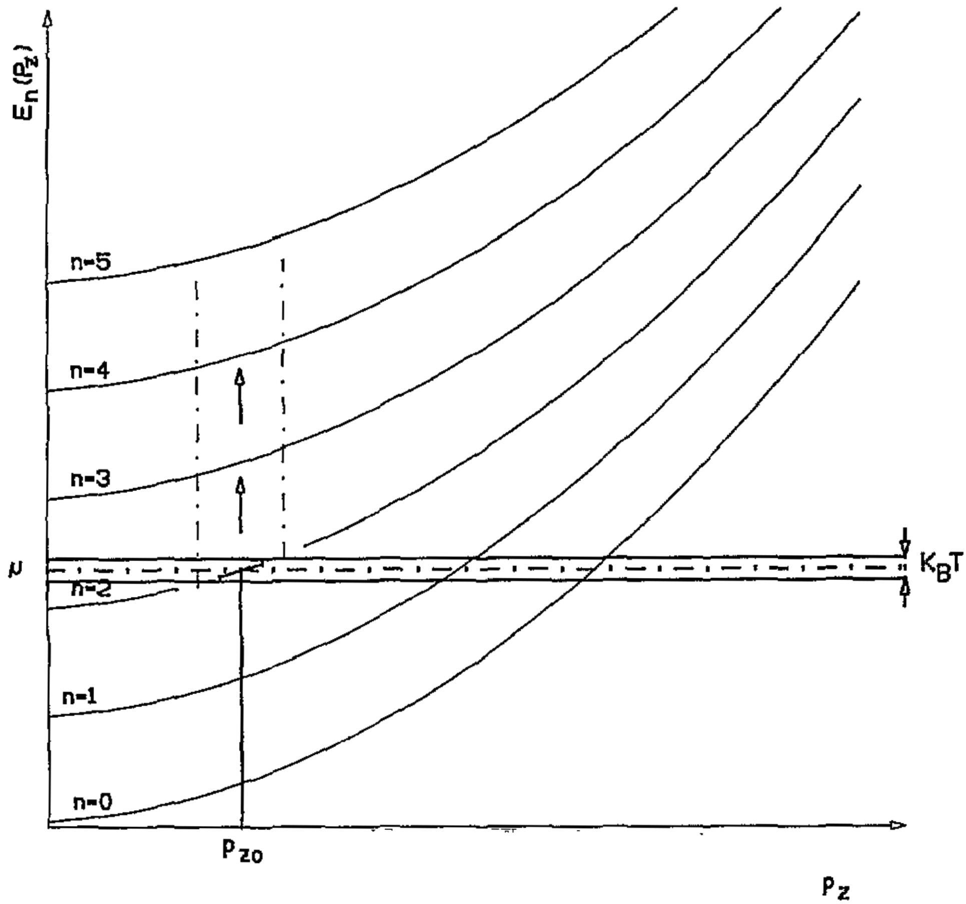

Figure 3. A uniform increase in hydrostatic pressure will force the small section of allowed states on the Landau level, in which the states with momentum p_{z0} are also members, to hop from one lower level to the next higher level as the Fermi level continuously rises in energy. This will eventually lead to giant oscillations of Γ_H.

magnetic field. Hence, unlike in the previous case, the quantized energy levels now remain unchanged indicating that the mechanism responsible for these oscillations is essentially different from that which is responsible for the oscillations predicted by GSF [1]. As already discussed in section 1, the electrons that effectively absorb the incident sound energy are those satisfying the condition

$$s = v_z \cos\theta \tag{19}$$

where $\theta \sim \pi/2$. These electrons form a very narrow belt on the Fermi surface and are characterized by the quasimomentum p_{z0}. If the position of the Fermi level (or $\sim$ chemical potential μ) is such that p_{z0} lies in the range of allowed values of p_z (these are the values of quasimomentum corresponding to a section on the Landau level found within the narrow strip of width $k_B T$ centred on the Fermi level (figure 3)), the ultrasonic absorption coefficient Γ_H will be relatively high. But now, if the hydrostatic pressure is further increased so that the Fermi level rises to a position where p_{z0} no longer lies in a range of allowed values of p_z, then Γ_H will be relatively small. This ultimately suggests that if the pressure is continuously and smoothly increased, the Fermi level will gradually sweep across the field of stationary Landau levels forcing the small section of states on the Landau level with z-momentum in the range of allowed values of p_z (in which p_{z0} is also a member) to hop from one lower Landau level to the next higher Landau level (figure 3). Since each Landau level is characterized by a singular density of states, which is given by [19]

$$\nu_n(\epsilon) = \frac{\hbar\omega}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left[\epsilon - \left(n + \frac{1}{2}\right)\hbar\omega\right]^{-1/2} \tag{20}$$

the hopping eventually leads to giant oscillations of the ultrasonic absorption coefficient Γ_H, as predicted in (15). The amplitude of these oscillations will continue to increase with pressure basically because of the fact that the number of electrons per unit volume, n_e, in the belt increases as the three-halves power of the Fermi energy, i.e.,

$$n_e \propto \epsilon_F^{3/2} \tag{21}$$

(in the nearly-free-electron model of the electron gas).

The fractional change in pressure ($\Delta P/P$) required to see these oscillations is $\sim \hbar\omega/\epsilon_F$, which, since $\hbar\omega \ll \epsilon_F$ (one of the conditions of GQO), is expected to be small.

As clearly shown in section 2, GQO of a second kind are expected to be seen for electron energies much higher than the threshold value Z^*. But a couple of technical points have to be observed before this could finally be achieved. Firstly, in order to avoid excessive compression of the lattice, the applied magnetic field has to be chosen such that it is only strong enough to inhibit the smoothing effect of temperature, which normally initiates interlevel transitions of electrons. This then means that, for lattice temperature of, say, $\sim$ 1.5 K, the applied field should be sufficiently larger than the required minimum value ($\sim$ 10 kG) so that the condition $\hbar\omega \gg k_B T$ is met, but it should not be too much in excess of $\sim$ 100 kG. Secondly, since the condition stated in (19) (sometimes referred to as the 'surf-riding condition') is derived from the conservation of energy and momentum principles under a very strict assumption that the phonon momentum $\hbar \boldsymbol{k}$ is much smaller than the Fermi momentum p_F, then in order to observe GQO of the absorption coefficient, the ultrasonic frequency has to be kept reasonably low.

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